

Mathematics: applications and interpretation
Higher level
Paper 1

Friday 6 May 2022 (afternoon)

Candidate session number

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2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

Please **do not** write on this page.

Answers written on this page
will not be marked.

25 pages

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Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 6]

A group of 130 applicants applied for admission into either the Arts programme or the Sciences programme at a university. The outcomes of their applications are shown in the following table.

	Accepted	Rejected
Arts programme	17	24
Sciences programme	25	64

- (a) Find the probability that a randomly chosen applicant from this group was accepted by the university. [1]

An applicant is chosen at random from this group. It is found that they were accepted into the programme of their choice.

- (b) Find the probability that the applicant applied for the Arts programme. [2]

Two different applicants are chosen at random from the original group.

- (c) Find the probability that both applicants applied to the Arts programme. [3]

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28EP02



28EP27

(Question 1 continued)

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28EP26



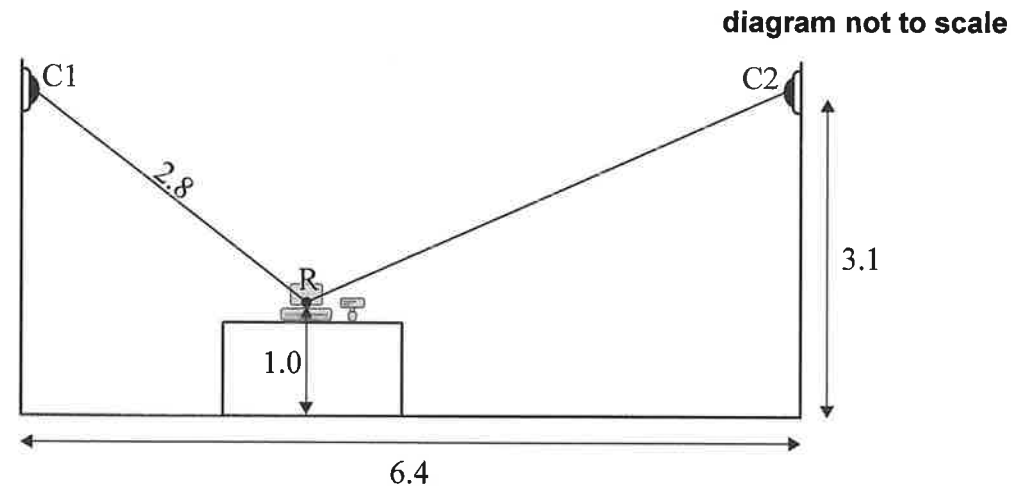
28EP03

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2. [Maximum mark: 8]

The owner of a convenience store installs two security cameras, represented by points C1 and C2. Both cameras point towards the centre of the store's cash register, represented by the point R.

The following diagram shows this information on a cross-section of the store.



The cameras are positioned at a height of 3.1 m, and the horizontal distance between the cameras is 6.4 m. The cash register is sitting on a counter so that its centre, R, is 1.0 m above the floor.

The distance from Camera 1 to the centre of the cash register is 2.8 m.

- Determine the angle of depression from Camera 1 to the centre of the cash register. Give your answer in degrees. [2]
- Calculate the distance from Camera 2 to the centre of the cash register. [4]
- Without further calculation, determine which camera has the largest angle of depression to the centre of the cash register. Justify your response. [2]

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(Question 17 continued)

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28EP06

16. [Maximum mark: 7]

The position vector of a particle, P , relative to a fixed origin O at time t is given by

$$\vec{OP} = \begin{pmatrix} \sin(t^2) \\ \cos(t^2) \end{pmatrix}.$$

(a) Find the velocity vector of P . [2]

(b) Show that the acceleration vector of P is never parallel to the position vector of P . [5]

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28EP23

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3. [Maximum mark: 7]

A polygraph test is used to determine whether people are telling the truth or not, but it is not completely accurate. When a person tells the truth, they have a 20% chance of failing the test. Each test outcome is independent of any previous test outcome.

10 people take a polygraph test and all 10 tell the truth.

- (a) Calculate the expected number of people who will pass this polygraph test. [2]
- (b) Calculate the probability that exactly 4 people will fail this polygraph test. [2]
- (c) Determine the probability that fewer than 7 people will pass this polygraph test. [3]

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28EP22



28EP07

Turn over

14. [Maximum mark: 4]

The shape of a vase is formed by rotating a curve about the y -axis.

The vase is 10 cm high. The internal radius of the vase is measured at 2 cm intervals along the height:

Height (cm)	Radius (cm)
0	4
2	6
4	8
6	7
8	3
10	5

Use the trapezoidal rule to estimate the volume of water that the vase can hold.



28EP20

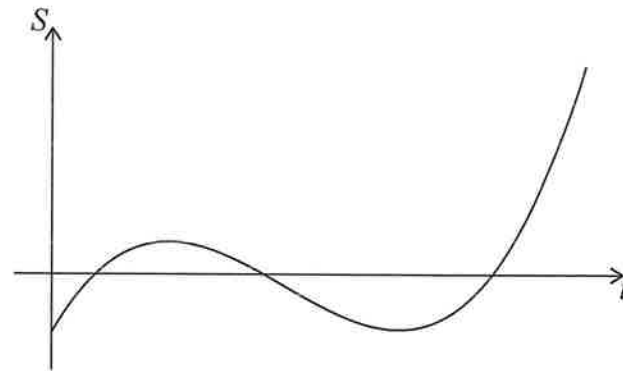
(Question 4 continued)



28EP09

5. [Maximum mark: 8]

The graph below shows the average savings, S thousand dollars, of a group of university graduates as a function of t , the number of years after graduating from university.



- (a) Write down one feature of this graph which suggests a cubic function might be appropriate to model this scenario.

[1]

The equation of the model can be expressed in the form $S = at^3 + bt^2 + ct + d$, where a , b , c and d are real constants.

The graph of the model must pass through the following four points.

t	0	1	2	3
S	–5	3	–1	–5

- (b) (i) Write down the value of d .
 (ii) Write down three simultaneous equations for a , b and c .
 (iii) Hence, or otherwise, find the values of a , b and c .

[4]

A negative value of S indicates that a graduate is expected to be in debt.

- (c) Use the model to determine the total length of time, in years, for which a graduate is expected to be in debt after graduating from university.

[3]

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28EP10

13. [Maximum mark: 5]

An electric circuit has two power sources. The voltage, V_1 , provided by the first power source, at time t , is modelled by

$$V_1 = \operatorname{Re}(2e^{3ti}).$$

The voltage, V_2 , provided by the second power source is modelled by

$$V_2 = \operatorname{Re}(5e^{(3t+4)i}).$$

The total voltage in the circuit, V_T , is given by

$$V_T = V_1 + V_2.$$

- (a) Find an expression for V_T in the form $A \cos(Bt + C)$, where A , B and C are real constants. [4]
 (b) Hence write down the maximum voltage in the circuit. [1]



28EP19

Turn over

12. [Maximum mark: 6]

The sex of cuttlefish is difficult to determine visually, so it is often found by weighing the cuttlefish.

The weights of adult male cuttlefish are known to be normally distributed with mean 10kg and standard deviation 0.5 kg.

The weights of adult female cuttlefish are known to be normally distributed with mean 12kg and standard deviation 1 kg.

A zoologist uses the null hypothesis that in the absence of information a cuttlefish is male.

If the weight is found to be above 11.5 kg the cuttlefish is classified as female.

- (a) Find the probability of making a Type I error when weighing a male cuttlefish. [2]
- (b) Find the probability of making a Type II error when weighing a female cuttlefish. [2]

90% of adult cuttlefish are male.

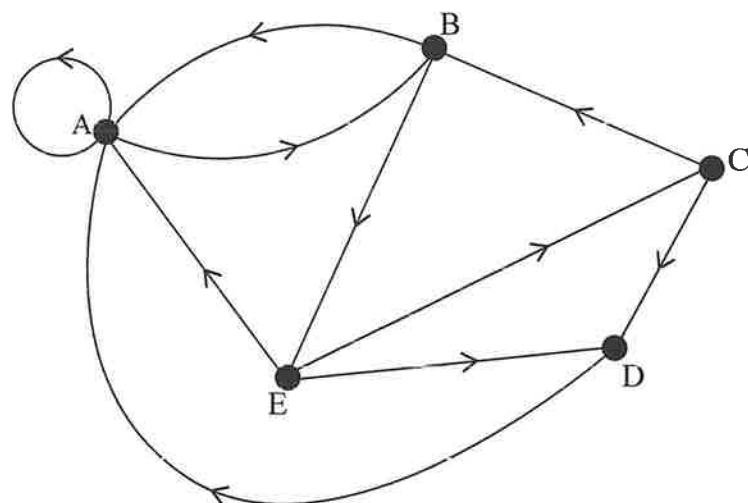
- (c) Find the probability of making an error using the zoologist's method. [2]

(Question 5 continued)

Turn over

6. [Maximum mark: 5]

Consider the following directed network.



- (a) Write down the adjacency matrix for this network. [2]
- (b) Determine the number of different walks of length 5 that start and end at the same vertex. [3]

11. [Maximum mark: 6]

Juliana plans to invest money for 10 years in an account paying 3.5% interest, compounded annually. She expects the annual inflation rate to be 2% per year throughout the 10-year period.

Juliana would like her investment to be worth a real value of \$4000, compared to current values, at the end of the 10-year period. She is considering two options.

Option 1: Make a one-time investment at the start of the 10-year period.

Option 2: Invest \$1000 at the start of the 10-year period and then invest \$ x into the account at the end of each year (including the first and last years).

- (a) For option 1, determine the minimum amount Juliana would need to invest. Give your answer to the nearest dollar. [3]
- (b) For option 2, find the minimum value of x that Juliana would need to invest each year. Give your answer to the nearest dollar. [3]

10. [Maximum mark: 5]

The function $f(x) = \ln\left(\frac{1}{x-2}\right)$ is defined for $x > 2$, $x \in \mathbb{R}$.

(a) Find an expression for $f^{-1}(x)$. You are not required to state a domain. [3]

(b) Solve $f(x) = f^{-1}(x)$. [2]

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28EP16

7. [Maximum mark: 5]

The sum of an infinite geometric sequence is 9.

The first term is 4 more than the second term.

Find the third term. Justify your answer.

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28EP13

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