Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 2

31 October 2023

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

2 hours

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

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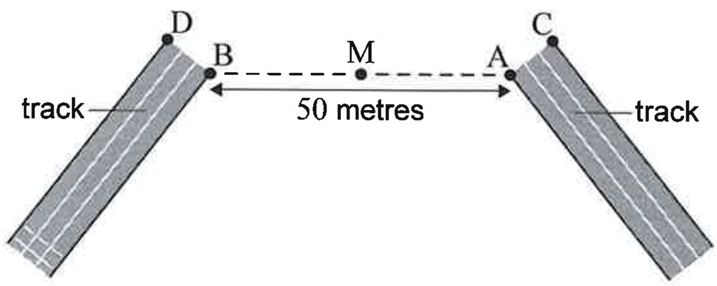
Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 15]

Madhu is designing a jogging track for the campus of her school. The following diagram shows an incomplete portion of the track.

Madhu wants to design the track such that the inner edge is a smooth curve from point A to point B, and the other edge is a smooth curve from point C to point D. The distance between points A and B is 50 metres.

diagram not to scale



To create a smooth curve, Madhu first walks to M, the midpoint of [AB].

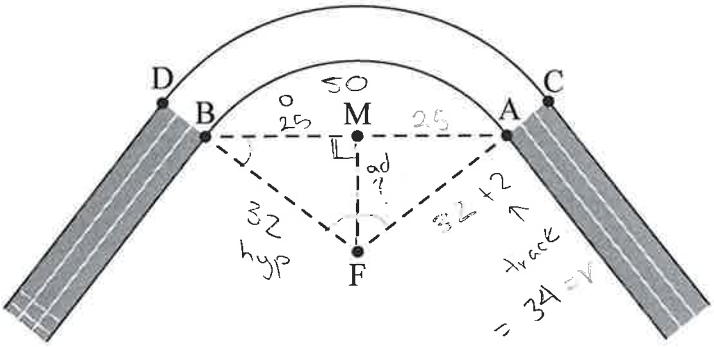
(a) Write down the length of [BM]. [1]

(This question continues on the following page)

(Question 1 continued)

Madhu then walks 20 metres in a direction perpendicular to [AB] to get from point M to point F. Point F is the centre of a circle whose arc will form the smooth curve between points A and B on the track, as shown in the following diagram.

diagram not to scale



(b) (i) Find the length of [BF].

(ii) Find $\text{B}\hat{\text{F}}\text{M}$. [4]

(c) Hence, find the length of arc AB. [3]

The outer edge of the track, from C to D, is also a circular arc with centre F, such that the track is 2 metres wide.

(d) Calculate the area of the curved portion of the track, ABDC. [4]

The base of the track will be made of concrete that is 12 cm deep.

(e) Calculate the volume of concrete needed to create the curved portion of the track. [3]

2. [Maximum mark: 18]

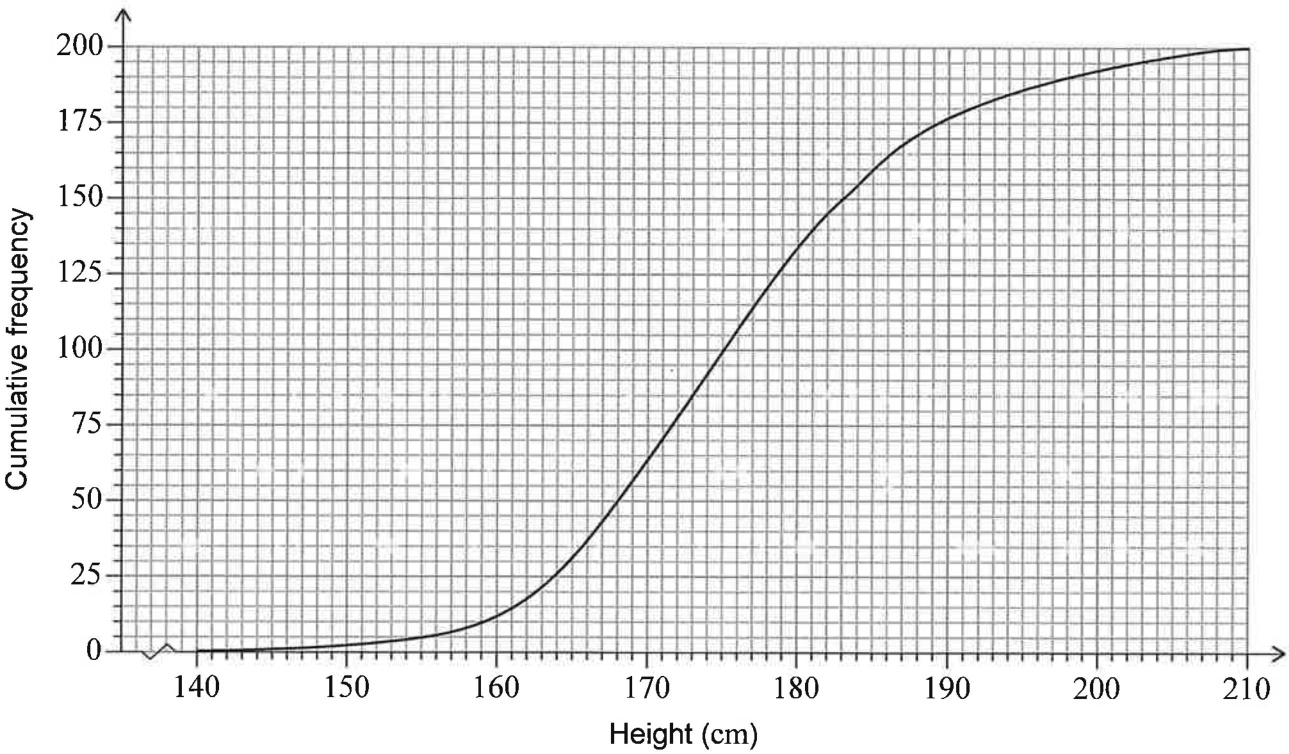
The heights, h, of 200 university students are recorded in the following table.

Height (cm)	**Frequency**
$140 \le h < 160$	11
$160 \le h < 170$	51
$170 \le h < 180$	68
$180 \le h < 190$	47
$190 \le h < 210$	23

(a) (i) Write down the mid-interval value of $140 \le h < 160$.

(ii) Calculate an estimate of the mean height of the 200 students. [3]

This table is used to create the following cumulative frequency graph.



(b) Use the cumulative frequency curve to estimate the interquartile range. [2]

Laszlo is a student in the data set and his height is 204 cm.

(c) Use your answer to part (b) to estimate whether Laszlo's height is an outlier for this data. Justify your answer. [3]

(This question continues on the following page)

(Question 2 continued)

It is believed that the heights of university students follow a normal distribution with mean $176\,\text{cm}$ and standard deviation $13.5\,\text{cm}$.

It is decided to perform a χ^2 goodness of fit test on the data to determine whether this sample of 200 students could have plausibly been drawn from an underlying distribution $\text{N}(176,\ 13.5^2)$.

(d) Write down the null and the alternative hypotheses for the test. [2]

As part of the test, the following table is created.

Height of student (cm)	**Observed frequency**	**Expected frequency**
$h < 160$	11	23.6
$160 \le h < 170$	51	42.1
$170 \le h < 180$	68	a
$180 \le h < 190$	47	46.7
$190 \le h$	23	b

(e) (i) Find the value of a and the value of b.

(ii) Hence, perform the test to a $5\,\%$ significance level, clearly stating the conclusion in context. [8]

3. [Maximum mark: 16]

Tiffany wants to buy a house for a price of 285 000 US Dollars (USD). She goes to a bank to get a loan to buy the house. To be eligible for the loan, Tiffany must make an initial down payment equal to 15 % of the price of the house.

The bank offers her a 30-year loan for the remaining balance, with a 4 % nominal interest rate per annum, compounded monthly. Tiffany will pay the loan in fixed payments at the end of each month.

(a) (i) Find the original amount of the loan after the down payment is paid. Give the exact answer.

(ii) Calculate Tiffany's monthly payment for this loan, to two decimal places. [5]

(b) Using your answer from part (a)(ii), calculate the total amount Tiffany will pay over the life of the loan, to the nearest dollar. Do **not** include the initial down payment. [2]

Tiffany would like to repay the loan faster and increases her payments such that she pays 1300 USD each month.

(c) Find the total number of monthly payments she will need to make to pay off the loan. [2]

This strategy will result in Tiffany's final payment being less than 1300 USD.

(d) Determine the amount of Tiffany's final payment, to two decimal places. [4]

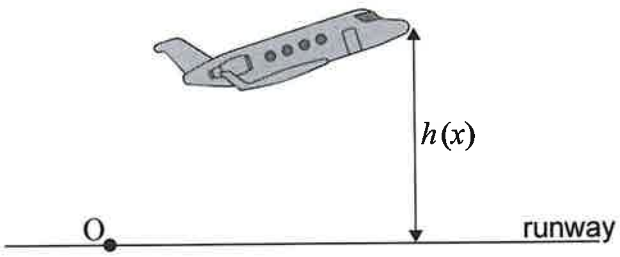
(e) Hence, determine the total amount Tiffany will save, to the nearest dollar, by making the higher monthly payments. [3]

4. [Maximum mark: 12]

A plane takes off from a horizontal runway. Let point O be the point where the plane begins to leave the runway and x be the horizontal distance, in km, of the plane from O. The function h models the vertical height, in km, of the nose of the plane from the horizontal runway, and is defined by

$$h(x) = \frac{10}{1+150\mathrm{e}^{-0.07x}} - 0.06,\ x \ge 0.$$

diagram not to scale



(a) (i) Find $h(0)$.

(ii) Interpret this value in terms of the context. [2]

(b) (i) Find the horizontal asymptote of the graph of $y = h(x)$.

(ii) Interpret this value in terms of the context. [2]

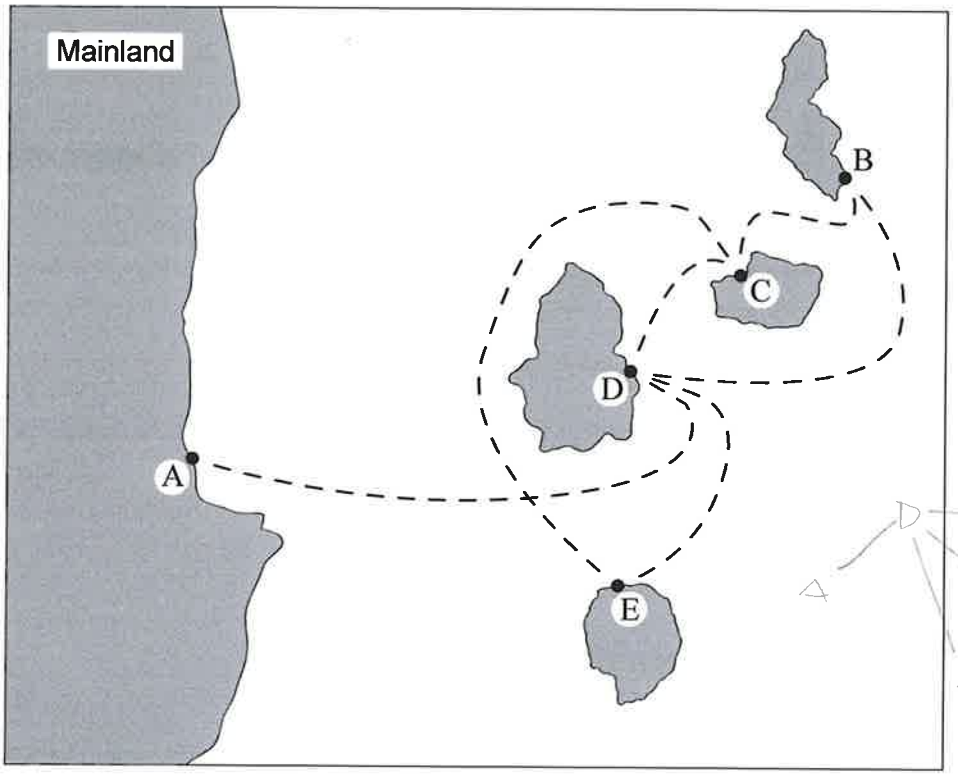
(c) Find $h'(x)$ in terms of x. [4]

A safety regulation recommends that $h'(x)$ never exceed 0.2.

(d) Given that this plane flies a distance of at least $200\,\mathrm{km}$ horizontally from point O, determine whether the plane is following this safety regulation. [4]

5. [Maximum mark: 18]

The following diagram is a map of a group of four islands and the closest mainland. Travel from the mainland and between the islands is by boat. The scheduled boat routes between the ports A, B, C, D and E are shown as dotted lines on the map.



Let the undirected graph G represent the boat routes between the ports A, B, C, D and E.

(a) Draw graph G. **[1]**

(b) Graph G can be represented by an adjacency matrix $\boldsymbol{P}$, where the rows and columns represent the ports in alphabetical order.

(i) Given that $\boldsymbol{P}^3 = \begin{pmatrix} 0 & 1 & 2 & 4 & 1 \\ 1 & 2 & 5 & a & 2 \\ 2 & 5 & 4 & 6 & 5 \\ 4 & a & 6 & 4 & 6 \\ 1 & 2 & 5 & 6 & 2 \end{pmatrix}$, find the value of a.

(ii) Hence, write down the number of different ways that someone could start at port B and end at port C, using three boat route journeys. **[3]**

(c) Find a possible Eulerian trail in G, starting at port A. **[2]**

(This question continues on the following page)

(Question 5 continued)

The cost of a journey on the different boat routes is given in the following table; all prices are given in USD. The cost of a journey is the same in either direction between two ports.

	A	B	C	D	E
A				10	
B			20	25	
C		20		50	45
D	10	25	50		30
E			45	30	

Sofia wants to make a trip where she travels on each of the boat routes at least once, beginning and ending at port A.

(d) Find the minimum cost of Sofia's trip. [3]

The boat company decides to add an additional boat route to make it possible to travel on each boat route **exactly** once, starting and ending at the same port.

(e) (i) Identify between which two ports the additional boat route should be added.

(ii) Determine the cost of the additional boat route such that the overall cost of the trip is the same as your answer to part (d). [2]

The boat company plans to redesign which ports are connected by boat routes. Their aim is to have a single boat trip that visits all the islands and minimizes the total distance travelled, starting and finishing at the mainland, A.

The following table shows the distances in kilometres between the ports A, B, C, D and E.

	A	B	C	D	E
A		80	90	50	60
B	80		30	70	120
C	90	30		45	100
D	50	70	45		55
E	60	120	100	55	

(f) (i) Use the nearest neighbour algorithm to find an upper bound for the minimum total distance.

(ii) Use the deleted vertex algorithm on port A to find a lower bound for the minimum total distance. [7]

6. [Maximum mark: 14]

François is a video game designer. He designs his games to take place in two dimensions, relative to an origin O. In one of his games, an object travels on a straight line L_1 with vector equation

$$\boldsymbol{r} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \end{pmatrix}.$$

(a) Write down L_1 in the form $x = x_0 + \lambda l$ and $y = y_0 + \lambda m$, where $l,\ m \in \mathbb{Z}$. [1]

François uses the matrix $\boldsymbol{T} = \begin{pmatrix} 1 & 7 \\ 7 & -1 \end{pmatrix}$ to transform L_1 into a new straight line L_2. The object will then travel along L_2.

(b) Find the vector equation of L_2. [4]

François knows that the transformation given by matrix $\boldsymbol{T}$ is made up of the following three separate transformations (in the order listed):

- A rotation of $\frac{\pi}{4}$, anticlockwise (counter-clockwise) about the origin O
- An enlargement of scale factor $5\sqrt{2}$, centred at O
- A reflection in the straight line $y = mx$, where $m = \tan\alpha$, $0 \le \alpha < \pi$

(c) Write down the matrix that represents

(i) the rotation.

(ii) the enlargement. [2]

(d) The matrix $\boldsymbol{R}$ represents the reflection. Write down $\boldsymbol{R}$ in terms of α. [1]

(e) Given that $\boldsymbol{T} = \boldsymbol{RX}$,

(i) use your answers to part (c) to find matrix $\boldsymbol{X}$.

(ii) hence, find the value of α. [6]

7. [Maximum mark: 17]

The city of Melba has an adult population of four million people. It is assumed that the weights of adults in Melba can be modelled by a normal distribution with mean 72 kg and standard deviation 10 kg.

(a) If 10 adults in Melba are chosen independently and at random, find the probability that more than 3 of them have a weight greater than 85 kg. [4]

Laetitia runs a travel agency in Melba. The elevator to her office is designed to hold a maximum of 8 people.

(b) Write down a probability distribution that models the total weight of 8 adults chosen independently and at random from Melba. [3]

The total weight of 8 adults exceeds w on less than 1% of all occasions that 8 adults enter the elevator.

(c) Find the value of w. [2]

A newspaper claims that 42% of the adults in Melba who go on holiday choose to go abroad. Laetitia believes that this is an overestimation of the true number. During the past month, Laetitia found that 67 of her clients chose a holiday abroad, and 133 chose a holiday that was not abroad.

(d) Laetitia decides to perform a test using the binomial distribution on her data for the population proportion, p, that go on holiday abroad.

(i) State **two** assumptions that Laetitia makes in order to conduct the test.

(ii) Write down the null and the alternative hypotheses for Laetitia's test, in terms of p.

(iii) Using the data from Laetitia's sample, perform the test at a 5% significance level to determine whether Laetitia's belief is reasonable. [8]