Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 3

29 October 2024

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

1 hour

302

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[55 marks]**.

B000

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Answer **both** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 24]

In this question you will use graph theory and transition matrices to solve problems about a manager visiting five factories.

Audrey is the quality control manager for a manufacturing company that has five factories, A, B, C, D and E.

She is planning a route to visit each factory once, starting and finishing from her home, H.

She determines the distance between each location, in kilometres, as shown in the table.

	A	B	C	D	E	H
A		100	150	70	50	40
B	100		80	140	130	85
C	150	80		90	120	160
D	70	140	90		60	100
E	50	130	120	60		70
H	40	85	160	100	70	

Audrey wants to find an upper and lower bound for the shortest total distance travelled on her route.

(a) Starting at H, use the nearest-neighbour algorithm to find an upper bound. [3]

To find a lower bound, Audrey uses the deleted vertex algorithm and deletes vertex H.

(b) (i) Use Prim's algorithm, starting at E, to find the weight of the minimum spanning tree for A, B, C, D and E. You should clearly state the order in which the edges are selected by the algorithm. [3]

(ii) Hence, find a lower bound. [2]

(This question continues on the following page)

(Question 1 continued)

After her initial visit to all factories, Audrey now decides she will visit one factory each day. She decides which factory to visit according to the following transition matrix, $\boldsymbol{T}$.

$$\boldsymbol{T} = \begin{array}{c} \\ \text{A} \\ \text{B} \\ \text{C} \\ \text{D} \\ \text{E} \end{array} \begin{array}{c} \begin{array}{ccccc} \text{A} & \text{B} & \text{C} & \text{D} & \text{E} \end{array} \\ \begin{pmatrix} 0 & 0.3 & 0.1 & 0 & 0.1 \\ 0 & 0 & 0.2 & 0.2 & 0.4 \\ 0 & 0.2 & 0.2 & p & 0.2 \\ 0.7 & 0.25 & q & r & 0 \\ 0.3 & 0.25 & 0.2 & 0.2 & 0.3 \end{pmatrix} \end{array}$$

After visiting factory D, there is a probability of 0.4 that Audrey will visit factory C next.

(c) Write down the value of

(i) p [1]

(ii) q [1]

(iii) r. [1]

(d) Audrey first visits factory A.

(i) Write down the initial state matrix, $\boldsymbol{S}_0$. [1]

(ii) Find the probability that the fifth factory that Audrey visits is C. [2]

(iii) Find the probability that the fifth factory that Audrey visits is the same as the second factory she visits. [5]

(e) Over a long period of time,

(i) find the proportion of Audrey's visits that are to factory A [2]

(ii) find the expected distance Audrey travels in a day, given that she always travels directly from home to a factory and then back home. [3]

2. [Maximum mark: 31]

In this question, you will first consider a statistical model for the number of fish caught in a lake and then consider a differential equation to model the growth of fish in the lake.

Althea enjoys fishing in a lake near her home. She usually fishes for 4 hours each day, and she records the length of each fish before releasing it back in the lake.

Althea decides to perform a χ^2 goodness of fit test, at the 5% significance level, to determine whether the number of fish caught can be modelled by a Poisson distribution.

She uses her records from the last 50 days to produce this table.

Number of fish caught per day	0	1	2	3	4	5	>5
Frequency	2	7	12	11	10	8	0

(a) Calculate the mean number of fish caught per day. [2]

(b) Hence, calculate the expected number of days, in a 50-day period, that Althea would catch no fish according to the Poisson model. [3]

Althea calculates the expected frequencies and records them in this table.

Number of fish caught per day	≤ 1	2	3	4	≥ 5
Expected frequency	10.9	11.6	11.2	8.05	8.25

(c) Explain why Althea has combined the columns for 0 and 1. [1]

(d) For this χ^2 goodness of fit test,

(i) write down a suitable null hypothesis [1]

(ii) write down the number of degrees of freedom [1]

(iii) find the p-value [3]

(iv) write down the conclusion, justifying your answer. [2]

(This question continues on the following page)

(Question 2 continued)

A factory is built on the edge of the lake, and Althea is concerned about the impact of pollution on the length of the fish. Based on her records, before the factory was built, the length of the fish could be modelled by a normal distribution with a mean of $50\,\text{cm}$.

After the factory is built, Althea collects a random sample of 15 fish and calculates that $\bar{x} = 48.6\,\text{cm}$ and $s_{n-1} = 2.56\,\text{cm}$.

Althea decides to test if there is sufficient evidence of a decrease in the population's mean length. The hypotheses for her test are H_0: $\mu = 50$ and H_1: $\mu < 50$.

(e) Perform the test, at the 5% significance level, to show that there is evidence of a decrease in the population's mean length. [3]

Althea now decides to consider the length of an individual fish in the lake as it grows. She researches the von Bertalanffy growth model, which states that

$$\frac{\mathrm{d}L}{\mathrm{d}t} = k(L_\infty - L),$$

where:

- L is the length of the fish (in centimetres) at time t (in months since birth)
- $L_\infty = \lim\limits_{t \to \infty} L$, the length the fish approaches as it grows
- k is a constant.

(f) Justifying your answer,

(i) state whether k is positive, negative, or could be either [2]

(ii) state the value of $\frac{\mathrm{d}L}{\mathrm{d}t}$ as $t \to \infty$ [2]

(iii) state a parameter in the model that could have been affected by the factory pollution in the lake. [2]

At $t = 0$, a fish has length $L_0\,\text{cm}$.

(g) Sketch a graph of $\frac{\mathrm{d}L}{\mathrm{d}t}$ against t. You should label the coordinates of any axes intercepts and the equation of any asymptotes on your sketch. [3]

(h) Solve the differential equation $\frac{\mathrm{d}L}{\mathrm{d}t} = k(L_\infty - L)$, using the initial condition. Write your answer in the form $L = f(t)$. [6]