Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 2

11 November 2025

Zone A morning | **Zone B** morning | **Zone C** morning

2 hours

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

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13 pages

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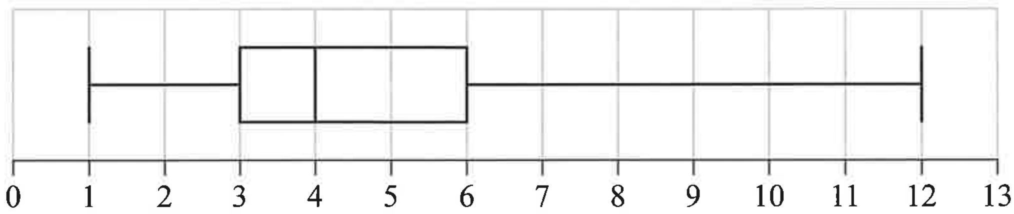
Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 15]

Dr Petrillo wrote a short scientific essay. He analysed the readability of his essay by counting the number of letters in each word.

Dr Petrillo constructs a box and whisker diagram for his data.



(a) Write down

(i) the median;

(ii) the upper quartile, Q_3;

(iii) the interquartile range, IQR. [3]

Dr Petrillo now wants to modify his diagram to show any outliers. He considers the longer words in his data and uses the following formula:

$$\text{outliers} > (1.5 \times \mathrm{IQR}) + \mathrm{Q}_3 .$$

Words with at least k letters are considered outliers.

(b) Find the value of k. [2]

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(Question 1 continued)

Dr Petrillo further considers the outliers and sees no reason to exclude them from his analysis.

The length of each word in the essay, n, and its associated frequency are given in the following table.

n	$1 \le n \le 2$	$3 \le n \le 4$	$5 \le n \le 6$	$7 \le n \le 8$	$9 \le n \le 10$	$11 \le n \le 12$	$n \ge 13$
Frequency	289	480	220	136	90	45	0

(c) Use the mid-interval values to calculate an estimate of the mean number of letters in each word. [3]

Dr Petrillo conducts a χ^2 goodness of fit test at the 1% significance level, to test the following null hypothesis:

H_0: The frequency of the number of letters in each word in his essay is consistent with the English language.

(d) Write down the alternative hypothesis for this test. [1]

The observed and expected frequencies of the number of letters in each word in his essay are listed in the following table.

n	$1 \le n \le 2$	$3 \le n \le 4$	$5 \le n \le 6$	$7 \le n \le 8$	$9 \le n \le 10$	$11 \le n \le 12$	$n \ge 13$
Observed frequency	289	480	220	136	90	45	0
Expected frequency	252.0	491.4	239.4	151.2	88.2	31.8	6.0

(e) (i) Write down the number of degrees of freedom.

(ii) Find the χ^2 statistic for this test.

(iii) Find the p-value for this test. [4]

The critical value for this test, at the 1% significance level, is 16.812.

(f) State whether Dr Petrillo should reject the null hypothesis. Justify your answer. [2]

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2. [Maximum mark: 16]

The height of a plant, h cm, depends on the time, t days, after it is planted.

For the first 15 days, the rate of change of the height of the plant is modelled as

$$\frac{dh}{dt} = 1.2 - 0.1t \text{, where } 0 \le t \le 15.$$

(a) (i) Find the value of $\frac{dh}{dt}$ at $t = 3$.

(ii) Interpret the meaning of your answer to part (a)(i) in context. [2]

(b) Use $\frac{dh}{dt}$ to find the value of t when the plant reaches its maximum height. [2]

Two days after it is planted, the height of the plant is 6.5 cm.

(c) Find an expression for h in terms of t, for $0 \le t \le 15$. [5]

(d) Hence, find the maximum height of the plant. [2]

A second plant was planted at the same time as the first plant. Its height, H cm, is modelled by $H = 2(1.2)^t$, where $0 \le t \le 15$.

(e) Write down the initial height of the second plant. [1]

Each day, the height of the second plant increases by p%.

(f) Find the value of p. [2]

(g) Find the value of t when the two plants have the same height. [2]

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3. [Maximum mark: 16]

Jenny designs a flexible suspension bridge, spanning 10 metres over the Swinburne Creek.

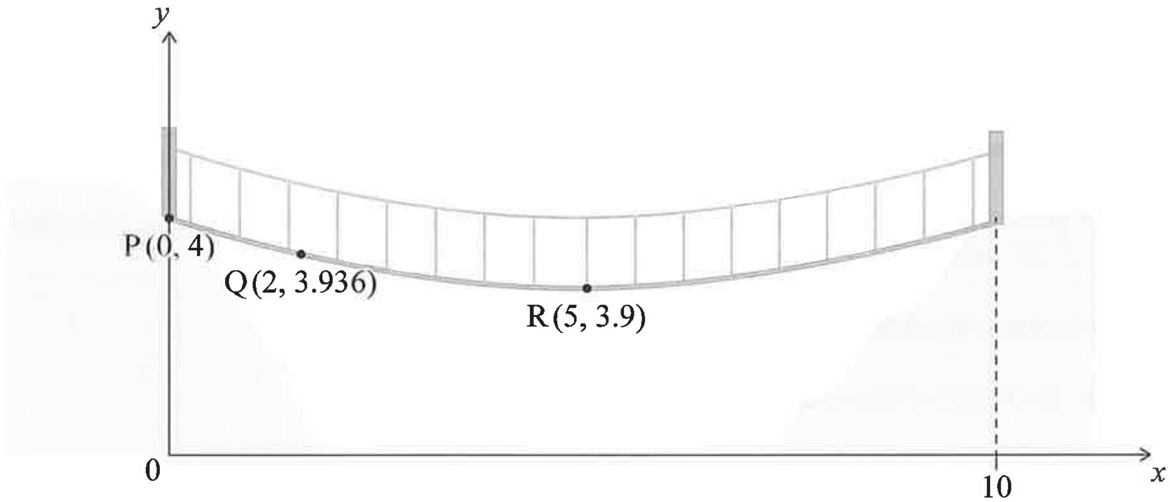
Jenny believes that when no people are on the bridge, the height, y, of the walkway on the bridge above the creek can be modelled by

$$y = ax^2 + bx + c \text{ for } 0 \le x \le 10.$$

She uses a coordinate system where the origin is directly below one side of the bridge, as shown in the following diagram. All distances are measured in metres.

According to this model, the points $\mathrm{P}(0, 4)$, $\mathrm{Q}(2, 3.936)$ and $\mathrm{R}(5, 3.9)$ will be located on the walkway of the bridge. Point R is the lowest point of the bridge.

diagram not to scale



(a) Write down the value of c, the y-intercept of the model. [1]

Using point Q, Jenny finds the equation $-0.064 = 4a + 2b$.

(b) (i) Find another equation in terms of a and b.

(ii) Hence or otherwise, find the value of a and the value of b. [4]

(c) For Jenny's model find

(i) $\dfrac{\mathrm{d}y}{\mathrm{d}x}$;

(ii) the steepest gradient of the walkway of bridge. [4]

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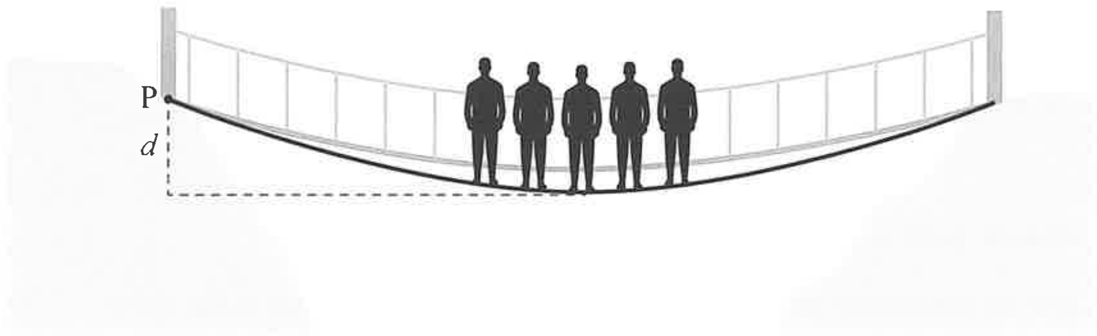
(Question 3 continued)

The deflection of the bridge, d, is the vertical distance of the walkway below P at the midpoint of the walkway.

After the bridge is built, Jenny explores the relationship between the number of people standing on the bridge and its deflection.

diagram not to scale



Jenny collected data on the number of people, n, on the bridge and the deflection, d metres, as shown in the following table.

Number of people, n	1	2	3	4	5	6
Deflection, d (m)	0.215	0.233	0.250	0.269	0.297	0.305

(d) For this data,

(i) find the value of Pearson's product-moment correlation coefficient, r;

(ii) comment on the strength of the correlation. [3]

The equation of the regression line for this data is $d = 0.01889n + 0.1954$ correct to four significant figures.

(e) Interpret, in context, the meaning of 0.01889 in the equation of the regression line. [1]

(f) Determine whether Jenny's **model** for the height of the walkway or the equation of the regression line gives the smaller estimate of the deflection. Justify your answer. [3]

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4. [Maximum mark: 14]

Peter likes to exercise and walks for a maximum of 12 hours each day. The distance he walks in one day, d kilometres, is modelled by

$$d = 3\ln(5t + 1), \text{ for } 0 \le t \le 12$$

where t is the time he spends walking, in hours, on that day.

(a) Find the time it takes for Peter to walk $6\,\text{km}$ in one day. [2]

Peter uses a motivational fitness app, and earns points each day for walking. The number of points earned, p, is given by

$$p = 20(1 - e^{-0.1d}), \text{ for } p \in \mathbb{R}.$$

(b) Find the number of points Peter earns for walking $6\,\text{km}$. [1]

(c) Find an expression for p as a function of t, giving your answer in the form

$$p = 20\left(1 - \frac{1}{(5t+1)^n}\right),$$

where n is a number to be determined. [4]

(d) Hence or otherwise, find the number of points Peter earns by walking for 2.5 hours. [1]

(e) Find the greatest number of points Peter can earn in one day. [2]

The fitness app introduces bonus points, b, for the total distance walked in one week, D, using

$$b = 50\left(1 - \frac{1}{D+1}\right).$$

Peter decides to walk for the same length of time, T hours, each day.

He wants to earn a total of 120 points in one week (7 days).

(f) Find the value of T. [4]

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5. [Maximum mark: 17]

A company sells spicy snacks. Over a long period of time, the company finds that 60% of people who taste the snacks will buy them. The company wants to test a new recipe to see if a greater percentage of people who taste these new snacks will buy them.

The company surveys the first 80 customers at a local supermarket who agree to taste the new snacks.

(a) State the name of the sampling method used. [1]

Out of the 80 people who taste the new snacks, 54 buy them. The company performs a hypothesis test to determine whether the new recipe is an improvement.

It assumes the responses of the 80 people are independent of each other. The null hypothesis is

$$\mathrm{H}_0 : p = 0.6,$$

where p is the proportion of people who buy the new snacks after tasting them.

(b) (i) Write down H_1, the alternative hypothesis.

(ii) Perform a suitable test, at the 5% significance level, stating the conclusion in context. [6]

(c) (i) Find the critical region for this test.

(ii) Given that $p = 0.65$, find the probability that the company makes a Type II error. [4]

During the survey, the company also recorded the age of each person. They want to find out if a person's age is independent of whether they will buy these snacks. The data for the 80 people is shown in the following table.

Age, x	**Will buy**	**Will not buy**
$x < 20$	22	8
$20 \le x < 30$	18	5
$30 \le x < 40$	6	8
$40 \le x$	8	5

(d) Perform a suitable test, at the 5% significance level, to determine if age is independent of whether someone will buy these snacks. [6]

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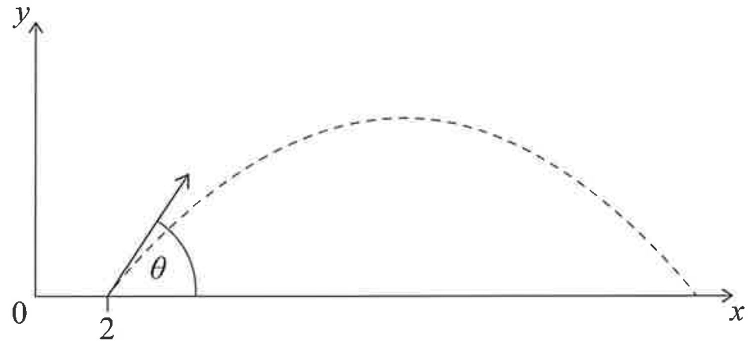
6. [Maximum mark: 14]

At an outdoor ballpark for children, a ball is launched from the ground, with initial speed $12\,\mathrm{m\,s^{-1}}$ and initial launch angle θ to the horizontal ground, where $0 < \theta < \frac{\pi}{2}$.

The position of the ball, t seconds after it is launched, is given by $\begin{pmatrix} x \\ y \end{pmatrix}$ where x is the horizontal displacement from an origin and y is the vertical displacement from the ground. Distances are measured in metres.

The ball's initial position is $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ and its path is shown in the diagram.



The ball can be modelled as a projectile, using $\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} 12\cos\theta \\ 12\sin\theta - 9.8t \end{pmatrix}$.

(a) Find an expression for x, in terms of θ and t. [3]

It is given that $y = (12\sin\theta)t - 4.9t^2$.

(b) When the ball hits the ground, show that $t = \frac{120\sin\theta}{49}$. [3]

Let x_g be the value of x when the ball hits the ground.

(c) Find an expression for x_g in terms of θ only. [2]

(d) Hence, find the value of θ which maximizes the value of x_g. [2]

The model is adapted to account for a horizontal wind with speed $3\,\mathrm{m\,s^{-1}}$.

The new model for the ball is $\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} 12\cos\theta - 3 \\ 12\sin\theta - 9.8t \end{pmatrix}$.

In the new model, the time taken for the ball to hit the ground remains $t = \frac{120\sin\theta}{49}$.

(e) (i) Find an expression for x_g in terms of θ only.

(ii) Hence, find the value of θ which maximizes the value of x_g. [4]

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7. [Maximum mark: 18]

The Fibonacci sequence has the initial values $0, 1, 1, 2, 3, 5, 8, \ldots$

To find the next number in the sequence, the previous two numbers are added. This can be written as $f_{n+1} = f_n + f_{n-1}$, together with $f_1 = 0$ and $f_2 = 1$.

Consecutive pairs in the sequence can be written as a matrix, $\begin{pmatrix} f_{n+1} \\ f_n \end{pmatrix}$, and used to produce a new sequence of matrices:

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \end{pmatrix}, \begin{pmatrix} 5 \\ 3 \end{pmatrix}, \ldots$$

(a) Write down the next two terms in this sequence of matrices. [2]

(b) Show that $\begin{pmatrix} f_{n+1} \\ f_n \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} f_n \\ f_{n-1} \end{pmatrix}$. [2]

(c) Show that the characteristic equation of $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$ is $\lambda^2 - \lambda - 1 = 0$. [2]

The eigenvalues of $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$ are λ_1 and λ_2, and a corresponding set of eigenvectors is $\begin{pmatrix} \lambda_1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} \lambda_2 \\ 1 \end{pmatrix}$.

(d) Using part (c), and without finding the value of λ_1, verify that $\begin{pmatrix} \lambda_1 \\ 1 \end{pmatrix}$ satisfies the equation for an eigenvector,

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ 1 \end{pmatrix} = \lambda_1 \begin{pmatrix} \lambda_1 \\ 1 \end{pmatrix}.$$ [2]

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(Question 7 continued)

It is given that $\begin{pmatrix} f_{n+1} \\ f_n \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^{n-1} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

The matrix $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^{n-1}$ can be written in the form $\boldsymbol{PD}^{n-1}\boldsymbol{P}^{-1}$, where $\boldsymbol{D}$ is a diagonal matrix.

(e) Write down, in terms of λ_1 and λ_2,

(i) $\boldsymbol{D}$;

(ii) $\boldsymbol{P}$;

(iii) $\boldsymbol{P}^{-1}$. [3]

(f) Hence, by multiplying the matrices $\boldsymbol{PD}^{n-1}\boldsymbol{P}^{-1}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, show that

$$f_n = \frac{1}{\lambda_1 - \lambda_2}\left((\lambda_1)^{n-1} - (\lambda_2)^{n-1}\right).$$ [4]

(g) (i) Use part (c) to find the exact value of λ_1 and λ_2.

(ii) Hence, write down an expression for f_n in terms of n. [3]

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